

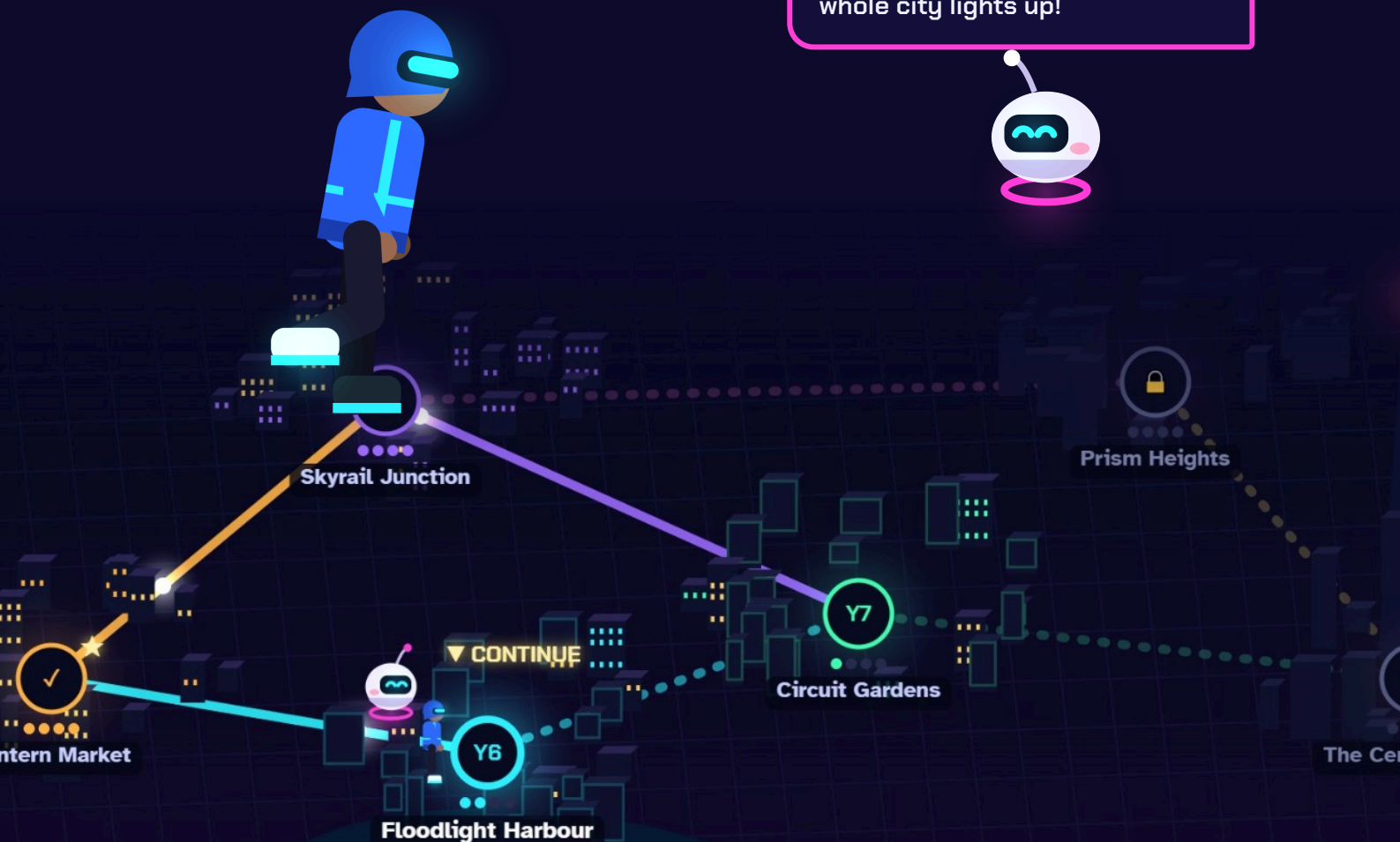
MATHS FOR YEARS 4–9

NEON // NUMBERS

Gridrunner's Study Guide

Everything you need to hack the city of Lumen back to life, with worked examples, practice and answers.

Every device in the city runs on maths. Learn the trick, and the whole city lights up!



SIX DISTRICTS · 30 STAGES · ONE CITY TO SAVE

Hi, Gridrunner!

This guide goes with the game **NEON // NUMBERS**. Each district of the city has its own kind of maths. For every district you'll find:

① Key ideas

The maths behind each device, with the same pictures MOCHI shows you in the game.

② MOCHI shows you

Worked examples, one step at a time.

③ Try it

Practice questions like the ones the devices ask. Write your answers on the lines.

④ Level up

A harder challenge for when you're ready. Answers are at the back, so no peeking!

MOCHI's three hints

Stuck at a terminal? Press **H** (or **Ask MOCHI**). Each press gives you a bit more help:

1

Nudge

A question to get you thinking.

2

Picture it

A diagram of *your* problem.

3

Worked example

The whole method, with different numbers.



MOCHI says: try to use as few hints as you can. Solve every device within two tries without the worked example, and you earn a **Mastery** ★ for that stage!

Gridrunner rules

- Mistakes are clues.** A wrong answer shows you what happened: the track is too short, the lift stops low, the fan freezes. Look at *how* it went wrong, then fix it.
- Say it before you press it.** "I think it's 6 because a quarter of 8 is 2..."
- Draw it.** Bars, number lines and grids make hard problems easy.
- Check it.** Put your answer back into the question. Does it work?
- Take your time.** The world freezes while a terminal is open, and nothing can hurt you there.

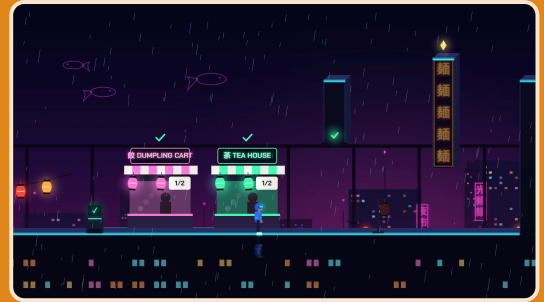
Controls

Move	Jump	Dash	Ground pound	Use terminal	Hint
← →	Space (hold = higher)	Shift	↓ in the air	E	H

Wall jump: jump at a wall, then jump again. Zipline: jump to grab the handle. Touch screens and gamepads work too.

Lantern Market

Power the night-market stalls, plot stepping stones across the rooftops and aim lasers in Laser Alley.



Fractions of a whole

Equivalent fractions

Fractions of amounts

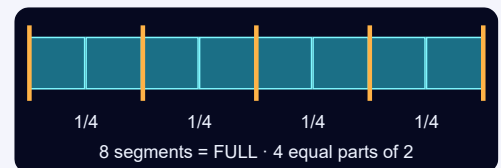
Coordinates

Angles

⚡ Fractions of a battery

The **bottom number** says how many equal parts the whole is split into. The **top number** says how many of those parts you take.

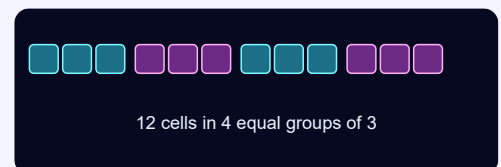
To find $\frac{3}{4}$ of an 8-segment battery: split 8 into 4 equal parts (2 each), then take 3 of them: **6 segments**.



🧱 Fractions of amounts

To share cells fairly, find **one part** first: divide by the bottom number. $\frac{1}{4}$ of 12 = $12 \div 4 = 3$.

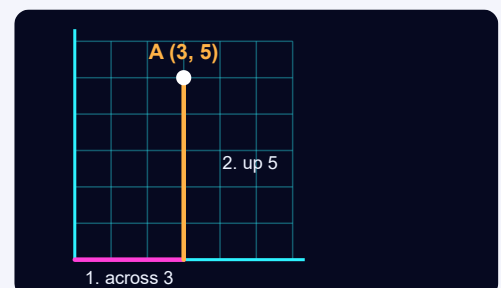
All the shares must add up to the whole amount.



📍 Coordinates

(x, y) means **across first, then up**. Think: "along the corridor, then up the stairs".

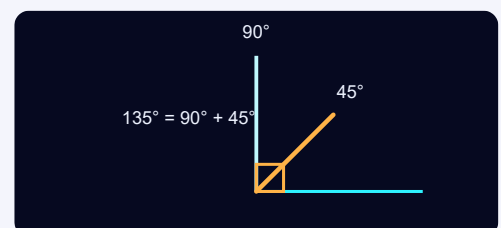
(3, 5) is 3 across and 5 up.



∠ Angles

A **right angle** is a quarter turn: 90° . Half a right angle is 45° . A straight line is 180° .

$1\frac{1}{2}$ right angles = $90^\circ + 45^\circ = 135^\circ$.



MOCHI shows you: sharing 12 cells $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$

1. $\frac{1}{2}$ of 12: $12 \div 2 = 6$
2. $\frac{1}{4}$ of 12: $12 \div 4 = 3$ (and the other $\frac{1}{4}$ is also 3)
3. Check: $6 + 3 + 3 = 12$ ✓ **Answer: 6, 3 and 3 cells.**

TRY IT**Lantern Market**

1. A battery has 4 segments. How many segments is $\frac{1}{2}$?

2. A battery has 8 segments. How many segments is $\frac{1}{4}$?

3. A battery has 12 segments. How many segments is $\frac{2}{3}$?

4. Share 20 cells between three stalls: $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. How many cells does each stall get?

5. Write the coordinates of the point that is 4 across and 2 up.

6. The laser must turn through $1\frac{1}{2}$ right angles. How many degrees is that?

★ Level up

An angle on a straight line is marked 130° . What is the angle next to it?



MOCHI says: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$. They're *equivalent*: different numbers, the same amount of battery!

Numerator the top number of a fraction

Equivalent worth the same amount

Right angle a quarter turn, 90°

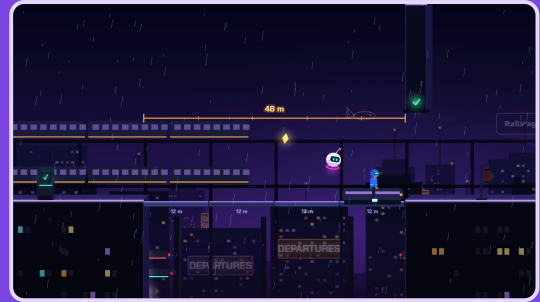
Denominator the bottom number: how many equal parts

Coordinates (across, up)

Degrees ($^\circ$) how we measure turns

Skyrail Junction

Rebuild the train tracks over the rooftops and print new blocks to fix the broken viaducts.



Dividing bigger numbers

Factor pairs

Area

Perimeter

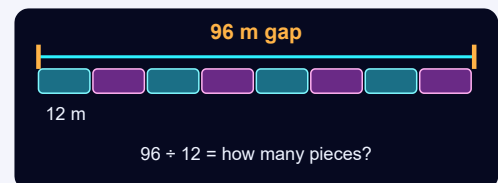
Decimals as fractions

= How many pieces fit?

A 96 m gap with 12 m rail pieces: $96 \div 12 = 8$ pieces.

Check: $8 \times 12 = 96$ ✓

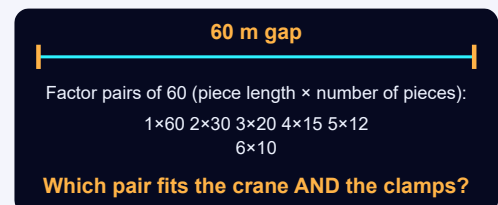
If it's too short, the shuttle gets stuck. If it's too long, it crashes!



Factor pairs

A **factor pair** is two numbers that multiply to make a number. For 60: 1×60 , 2×30 , 3×20 , 4×15 , 5×12 , 6×10 .

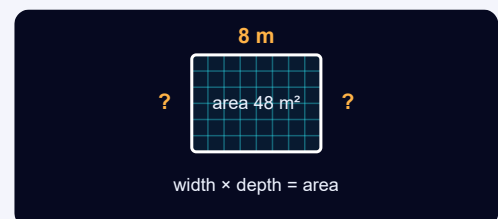
When the crane and clamps have limits, only some pairs will work.



□ Area and perimeter

Area = width × depth (the squares inside). **Perimeter** = the distance all the way round.

Area 48 m^2 , width 8 m: depth = $48 \div 8 = 6$ m.



MOCHI shows you: a factor-pair track

1. The gap is 60 m. Pieces can be up to 12 m, and there are at most 6 clamps.
2. Factor pairs of 60: 1×60 , 2×30 , 3×20 , 4×15 , 5×12 , 6×10 .
3. Piece ≤ 12 m and pieces ≤ 6 : **12 m × 5 or 10 m × 6**. Both work!

MOCHI shows you: perimeter

1. The safety tape round a hole is 30 m, and the hole is 9 m wide.
2. Take away the two widths: $30 - 9 - 9 = 12$ m for the two depths.
3. $12 \div 2 = 6$ m deep.

TRY IT**Skyrail Junction**

1. $144 \div 12 =$

2. A 120 m gap with 15 m pieces. How many pieces?

3. List all the factor pairs of 36.

4. A hole has an area of 56 m^2 and is 8 m wide. How deep is it?

5. The tape round a hole is 26 m, and the hole is 8 m wide. How deep is it?

6. A battery has 20 segments. How many segments is 0.35?

★ Level up

A 240 m gap. The crane lifts pieces up to 25 m and there are at most 12 clamps. Which piece lengths work?



MOCHI says: $0.5 = \frac{1}{2}$, $0.25 = \frac{1}{4}$, $0.75 = \frac{3}{4}$ and $0.1 = \frac{1}{10}$. Decimals are just fractions in disguise!

Factor a number that divides exactly into another

Area the space inside a shape (m^2)

Decimal a number with a decimal point, like 0.75

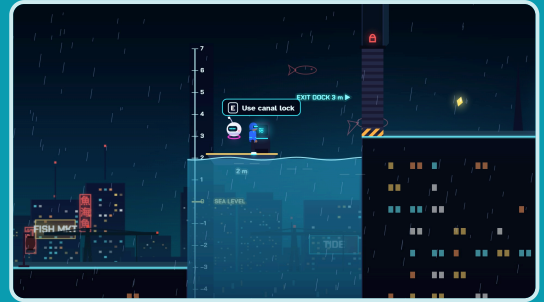
Factor pair two factors that multiply to make the number

Perimeter the distance all the way round (m)

Hundredths 0.01, one part in a hundred

Floodlight Harbour

Raise and lower the canal locks around sea level, and sneak past the searchlights on the docks.



Negative numbers

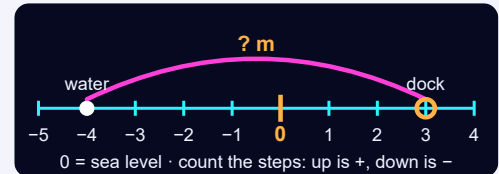
Four-quadrant coordinates

Percentages

Comparing probabilities

≈ Across zero

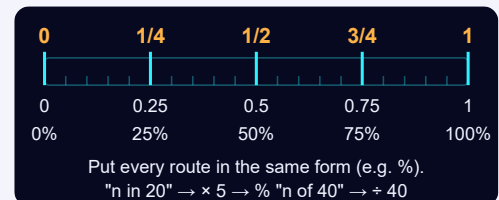
Sea level is 0. Below it the numbers are negative. Count the steps from the water to the dock: **up is +, down is -**.
Crossing zero? Split the journey at 0.



🎯 Which is least likely?

Change every chance into the **same form**, usually a percentage, then compare.

$$0.4 = 40\% \cdot 3 \text{ in } 20 = 15\% \cdot 14 \text{ of } 40 = 35\%$$



⌘ Four quadrants and percentages

In coordinates, a negative x means go **left**, and a negative y means go **down**. $(-2, 3)$ is 2 left, then 3 up.

For percentages, find 10% first ($\div 10$), then build up: 35% of 40 = 30% (12) + 5% (2) = **14**.

MOCHI shows you: the canal lock

1. The water is at -4 m. The exit dock is at 3 m.
2. From -4 up to 0 is $+4$. From 0 up to 3 is $+3$.
3. $+4 + 3 = +7$ m (pump the water up 7 m). Check: $-4 + 7 = 3$ ✓

MOCHI shows you: going down

1. Water at 5 m, dock at -2 m.
2. From 5 down to 0 is -5 . From 0 down to -2 is -2 .
3. Total -7 m (let the water down 7 m).

TRY IT**Floodlight Harbour**

1. The water is at -3 m and the dock is at 4 m. What change do you need?

2. The water is at 2 m and the dock is at -5 m. What change do you need?

3. The water is at -6 m and the dock is at -1 m. What change do you need?

4. Describe how to plot $(-2, 3)$ from the origin.

5. What is 35% of 40 ?

6. Which searchlight is on least often: 0.4 , 3 in 10 , or 45% ?

★ Level up

The water is 4.5 m below sea level. The dock is 2 m above sea level. What change do you need?



MOCHI says: a bigger number at the bottom doesn't mean a bigger chance! 3 in 20 is smaller than 1 in 4 .

Negative number a number below zero, like -4

Quadrant one of the four quarters of a coordinate grid

Probability how likely something is, from 0 to 1

Sea level zero on the lock gauge

Percentage out of 100 (%)

Origin the point $(0, 0)$

Circuit Gardens

Mix coolant to restart the giant fans and ride their updrafts.
Balance the see-saw bridges with equations.



Ratio

Percentages of amounts

Two-step equations

Brackets

Angle facts

Sharing in a ratio

A ratio 2 : 3 means **5 equal parts** (2 + 3). Find one part, then multiply.

20 L in the ratio 2 : 3 → $20 \div 5 = 4$ L per part → **8 L and 12 L**.

whole tank = 20 L



concentrate 2 boxes · water 3 boxes

each box = $20 \div 5$ L

Balance the bridge

An equation is a balance. Whatever you do to one side, **do to the other**.

$2x + 7 = 21$ → take 7 from both sides → $2x = 14$ → $x = 7$.

 $2x + 7 = 21$ 

Do the same to both sides
so the bridge stays balanced.

Angle facts

Angles on a **straight line** add to 180° . Angles in a **right angle** add to 90° . Angles **around a point** add to 360° .

MOCHI shows you: one part known

1. The mix is concentrate : water = 3 : 4. You pour in 9 L of concentrate.
2. 3 parts = 9 L, so 1 part = 3 L.
3. Water is 4 parts: $4 \times 3 = 12$ L.

MOCHI shows you: brackets

1. $3(x + 2) = 21$
2. Divide both sides by 3: $x + 2 = 7$
3. Take 2 from both sides: **$x = 5$** . Check: $3 \times (5 + 2) = 21$ ✓

TRY IT**Circuit Gardens**

1. Share 30 L in the ratio 1 : 2.

2. Share 28 L in the ratio 3 : 4.

3. The ratio is 2 : 5 and you have 6 L of concentrate. How much water do you need?

4. What is 25% of 40 L?

5. Solve $4x + 3 = 31$.

6. Solve $2(x + 4) = 22$.

7. One angle around a point is 250° . What is the other angle?

★ Level up

Solve $5x + 7 = 42$. Then find the angle that goes with 35° to make a right angle.

Ratio compares amounts, like 2 : 3

Equation two sides that are equal

Brackets work out what's inside first

Part one equal share of a ratio

Unknown the letter we're finding, like x

Complementary two angles that make 90°

Prism Heights

Program the lifts in the glass towers, transform blueprints onto their anchors and balance equation bridges.



Rates

Gradients

When two lines meet

Transformations

Unknowns on both sides

↑ Speed and gradient

Speed = distance ÷ time. 10 m in 4 s = 2.5 m/s.

On a height–time graph, speed is the **gradient**: rise ÷ run between two points.



Twin lifts: when are they level?

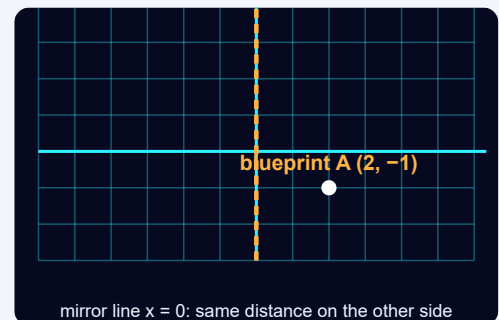
Lift A: $h = 2t$. Lift B: $h = 5 + t$. They're level when the heights are **equal**:

$2t = 5 + t \rightarrow t = 5$ seconds (both at 10 m).



⇌ Transformations

Translate: add the vector. **Reflect:** the same distance on the other side of the mirror line. **Rotate 180°** about (0, 0): $(x, y) \rightarrow (-x, -y)$.



MOCHI shows you: unknowns on both sides

- $5x + 3 = 2x + 18$
- Take $2x$ from both sides: $3x + 3 = 18$
- Take 3 from both sides: $3x = 15$, so $x = 5$. Check: $28 = 28$ ✓

TRY IT**Prism Heights**

1. A lift climbs 12 m in 3 s. What is its speed?

2. A lift climbs at 2.5 m/s. How long does it take to rise 10 m?

3. Find the gradient of the line through (2, 3) and (4, 9).

4. Lift A: $h = 3t$. Lift B: $h = 6 + t$. When are they level, and how high?

5. Reflect (3, -2) in the y-axis (the line $x = 0$).

6. Rotate (-4, 1) by 180° about the origin.

7. Solve $7x - 4 = 3x + 12$.

★ Level up

Solve $4(x + 2) = x + 23$. Then give the vector that moves (-3, 1) to (1, -1).

Rate how fast something changes, like m/s

Vector a movement (across, up)

Rotation a turn about a point

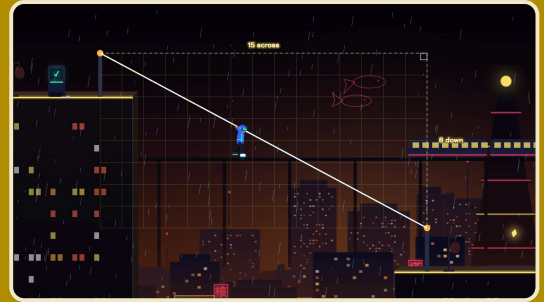
Gradient steepness: $\text{rise} \div \text{run}$

Reflection a flip in a mirror line

Image where a shape ends up after a transformation

The Central Spire

Zipline between rooftops, cross the security pattern bridges and face the Chairman at the top of the Spire.



Pythagoras' theorem

Sequences and the nth term

Combined probability

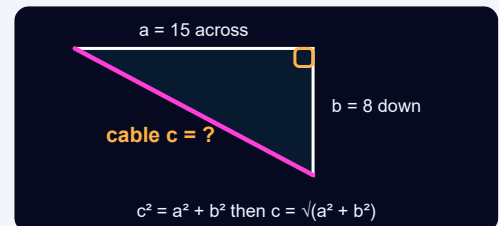
Indices

Trigonometry

/ Pythagoras

In a right-angled triangle, the longest side (the hypotenuse) is **c**: $c^2 = a^2 + b^2$.

15 across, 8 down: $225 + 64 = 289$, and $\sqrt{289} = 17$ m of cable.



... Sequences and the nth term

An nth-term rule makes the sequence: put in $n = 1, 2, 3, \dots$

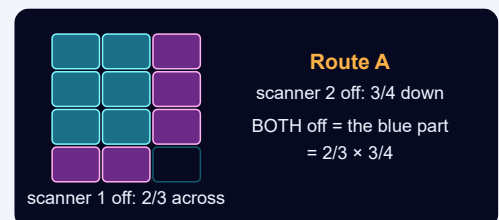
$3n + 1 \rightarrow 4, 7, 10, 13, \dots$ Only those planks are real!



🕒 Both scanners off

For two things to happen together, **multiply** the probabilities: off $\frac{2}{3}$ and off $\frac{3}{4} \rightarrow \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

Indices: $2^{-3} = 1/2^3 = \frac{1}{8}$. **Trig:** $\tan(\text{angle}) = \text{rise} \div \text{run}$.



MOCHI shows you: a cable to 1 decimal place

- 14 across and 6 down.
- $c^2 = 14^2 + 6^2 = 196 + 36 = 232$
- $c = \sqrt{232} = 15.23\dots \approx 15.2$ m.

TRY IT**The Central Spire**

1. A cable runs 6 m across and 8 m down. How long is it?

2. A cable runs 5 m across and 12 m down. How long is it?

3. A cable runs 9 m across and 4 m down. How long is it, to 1 decimal place?

4. Write the first four terms of $4n - 1$.

5. Is 50 in the sequence $3n + 2$? How do you know?

6. Two scanners are off $\frac{1}{2}$ and $\frac{4}{5}$ of the time. What is the chance that both are off?

7. Work out 2^{-3} .

8. A laser must rise 4 m over a run of 4 m. What angle is that?

★ Level up

A cable goes from anchor $(-4, 20)$ to anchor $(8, 11)$. How long is it? And what is the n th term of 5, 9, 13, 17, ...?

Hypotenuse the longest side, opposite the right angle

n th term a rule for any term in a sequence

Index / power how many times to multiply: $2^3 = 8$

Square root ($\sqrt{}$) undoes squaring: $\sqrt{289} = 17$

Independent events that don't affect each other

tan rise $\div$ run in a right-angled triangle

Answers

Lantern Market

1. 2
2. 2
3. 8
4. 10, 5 and 5
5. (4, 2)
6. 135°

Level up: 50° ($180 - 130$)

Skyrail Junction

1. 12
2. 8
3. 1×36 , 2×18 , 3×12 , 4×9 , 6×6
4. 7 m
5. 5 m ($26 - 16 = 10$, then $\div 2$)
6. 7

Level up: $20 \text{ m} \times 12 \text{ pieces}$ or $24 \text{ m} \times 10 \text{ pieces}$

Floodlight Harbour

1. +7 m
2. -7 m
3. +5 m
4. 2 left, then 3 up
5. 14
6. 3 in 10 (30%)

Level up: +6.5 m

Circuit Gardens

1. 10 L and 20 L
2. 12 L and 16 L
3. 15 L
4. 10 L
5. $x = 7$
6. $x = 7$
7. 110°

Level up: $x = 7$; 55°

Prism Heights

1. 4 m/s
2. 4 s
3. 3 (rise $6 \div$ run 2)
4. $t = 3 \text{ s}$, both at 9 m
5. (-3, -2)
6. (4, -1)
7. $x = 4$

Level up: $x = 5$; vector (4, -2)

The Central Spire

1. 10 m
2. 13 m
3. 9.8 m ($\sqrt{97}$)
4. 3, 7, 11, 15
5. Yes: $3n + 2 = 50$ gives $n = 16$
6. $\frac{2}{5}$ (40%)
7. $\frac{1}{8}$
8. 45°

Level up: 15 m (12 across, 9 down); $4n + 1$



MOCHI says: got one wrong? Brilliant, now you know what to practise! Find the key idea for that district and try the question again.

My progress

Tick each stage when you restore it, and colour in the star when you earn its Mastery ★. Every stage makes new numbers each time you play, so you can practise as often as you like!

District	1 · First stage	2 · Challenge	3 · Challenge	4 · Boss	★ Remix
Lantern Market	☐ ★	☐ ★	☐ ★	☐ ★	☐ ★
Skyrail Junction	☐ ★	☐ ★	☐ ★	☐ ★	☐ ★
Floodlight Harbour	☐ ★	☐ ★	☐ ★	☐ ★	☐ ★
Circuit Gardens	☐ ★	☐ ★	☐ ★	☐ ★	☐ ★
Prism Heights	☐ ★	☐ ★	☐ ★	☐ ★	☐ ★
The Central Spire	☐ ★	☐ ★	☐ ★	☐ ★	☐ ★

District	Stages
Lantern Market	Power to the Stalls · Rooftop Relay · Laser Alley · The Billboard Warden · Lantern Remix
Skyrail Junction	Signal Restart · Rooftop Landings · Express Line · The Timetable · Junction Remix
Floodlight Harbour	High Tide · Rescue Routes · Searchlight Docks · The Harbourmaster · Harbour Remix
Circuit Gardens	Coolant Crisis · Vertical Garden · Balance Walk · Overgrowth · Garden Remix
Prism Heights	Lift Logic · Mirror Halls · Equation Bridges · The Prism Engine · Prism Remix
The Central Spire	Rooftop Crossings · Security Patterns · Core Ascent · The Chairman · Spire Remix

Glossary

Area: the space inside a flat shape, measured in squares (m²).

Coordinates: a pair (x, y) giving a position: across, then up.

Decimal: a number with tenths, hundredths..., like 0.35.

Denominator: the bottom number of a fraction.

Equation: a statement that two things are equal, like $3x + 5 = 26$.

Equivalent fractions: fractions worth the same, like $\frac{1}{2}$ and $\frac{4}{8}$.

Factor: a number that divides exactly into another.

Gradient: how steep a line is: rise ÷ run.

Hypotenuse: the longest side of a right-angled triangle.

Index (power): how many times a number is multiplied by itself.

Negative number: a number less than zero.

nth term: a rule that gives any term of a sequence.

Percentage: a number out of 100.

Perimeter: the distance all the way round a shape.

Probability: how likely something is, from 0 (impossible) to 1 (certain).

Pythagoras' theorem: $c^2 = a^2 + b^2$ in a right-angled triangle.

Rate: how one amount changes compared with another, like metres per second.

Ratio: a comparison of amounts, like 2 : 3.

Reflection: a flip in a mirror line.

Right angle: a quarter turn, 90°.

Rotation: a turn about a fixed point.

Sequence: numbers that follow a rule.

Translation: a slide, with no turning or flipping.

Trigonometry: using ratios like tan to find angles and sides.